Experiment No : M7

Experiment Name: Moments of Inertia and Body Shape

Objective:

1. Comparing the periods of oscillation of two bodies having different masses, but the same moment of inertia.
2. Comparing the periods of oscillation of hollow bodies and solid bodies having the same mass and the same dimensions.
3. Comparing the periods of oscillation of two bodies having the same mass and the same body shape, but different dimensions.

Keywords: Rotational motions, moment of inertia, oscillation, period.

Theoretical Background:

The moment of inertia is a measure of the resistance of a body against a change of its rotational motion and it depends on the distribution of its mass relative to the axis of rotation. For a calculation of the moment of inertia I, the body is subdivided into sufficiently small mass elements Δm_i with distances r_i from the axis of rotation and a sum is taken over all mass elements:

$$I = \sum_i \Delta\, m_i r_i^2 \qquad 7.1$$

For bodies with a continuous mass distribution, the sum can be converted into an integral. If, in addition, the mass distribution is homogeneous, the integral reads

$$I = M \frac{1}{V} \int_V r^2\, dV \qquad 7.2$$

where M is total mass, V is total volume and r is the distance of a volume element dV from the axis of rotation.

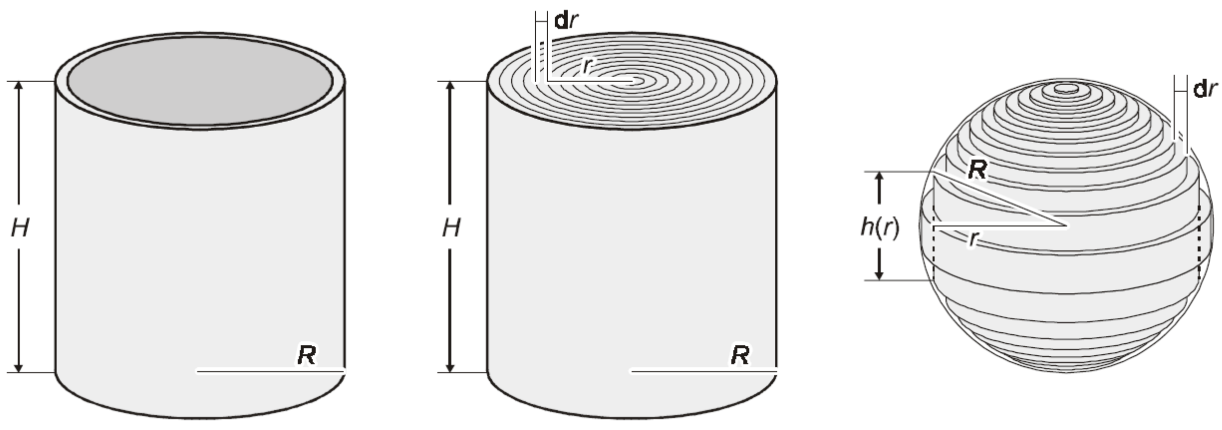

Figure 7.1: Calculation of the moments of inertia of a hollow cylinder, a solid cylinder and a sphere

The calculation of the integral is simplified when rotationally symmetric bodies are considered which rotate around their axis of symmetry. The simplest case is that of a hollow cylinder with radius R. As all mass elements have the distance R from the axis of rotation, the moment of inertia of the hollow cylinder (HC) is

$$I_{HC} = MR^2 \quad 7.3$$

In the case of a solid cylinder (*SC*) with equal mass *M* and equal radius *R*, Eq. 7.2 leads to the formula

$$I_{SC} = M\frac{1}{V}\int_0^R r^2\, 2\pi r H dr \quad 7.4$$

with $V = \pi R^2 H$and the result is

$$I_{SC} = \frac{1}{2}MR^2 \quad 7.5$$

That means, the moment of inertia of a solid cylinder is smaller than that of the hollow cylinder as the distances of the mass elements from the axis of rotation are between 0 and *R*. An even smaller value is expected for the moment of inertia of a solid sphere (*SS*) with radius *R* (see Fig. 7.1). In this case, Eq. 7.2 leads to the formula

$$I_{SS} = M\frac{1}{V}\int_0^R r^2\, 2\pi r 2\sqrt{R^2 - r^2} dr \quad 7.6$$

with $V = \frac{4}{3}\pi R^3$and the result is

$$I_{SS} = \frac{2}{5}MR^2 \quad 7.7$$

Thus, apart form the mass *M* and the radius *R* of the bodies under consideration a dimensionless factor enters the calculation of the moment of inertia, which depends on the shape of the respective body.

The moment of inertia is determined from the period of oscillation of a torsion axle, on which the test body is fixed and which is connected elastically to the stand via a helical spring. The system is excited to perform harmonic oscillations. If the restoring torque *D* is known, the moment of inertia of the test body is calculated from the period of oscillation *T* according to

$$I = D\left(\frac{T}{2\pi}\right)^2 \quad 7.8$$